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Inverse Problem of Estimating Interface Conductance Between Periodically Contacting Surfaces

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The conjugate gradient method of minimization with adjoint equation is used to solve the inverse problem of estimating the timewise variation of interface conductance between periodically contacting solids, under quasi-steady-state conditions. It is assumed that no prior information is available on the functional form of the interface conductance, except the magnitude of the period. The accuracy of the inverse analysis is examined by using simulated inexact temperature measurements obtained at the interior of the region. Small periods are usually the most difficult on which to perform an inverse analysis. For such cases, the present method is found to be more accurate and stable than the B-Spline approach.

Nomenclature

e = error
e_{total} = error defined by Eq. (23)
h = interface conductance
J = functional defined by Eq. (4)
J' = gradient of the functional defined by Eq. (16)
K = thermal conductivity
L = length
N_i = number of sensors in region i
p = direction of descent defined by Eq. (5b)
T = temperature
T_{var} = maximum temperature variation in the regions
t = time variable
x = spatial variable
Y = measured temperature
α = thermal diffusivity
β = search step size defined by Eq. (9)
Δ = variation
Δ_i = distance between sensors in region i
ΔT_i = sensitivity function for region i
δ_i = distance of the first sensor to the interface in region i
δ_T = thermal layer
γ = conjugation coefficient defined by Eq. (5c)
ε, ε_δ = tolerance and tolerance for the thermal layer, respectively
λ_i = Lagrange multiplier for region i
σ = standard deviation of the measurements
τ = period
ω = random variable used in Eq. (22)

Subscripts
k = number of iterations
0 = refers to the noncontact end of the regions
1, 2 = regions 1 and 2, respectively

Superscripts
k = number of iterations
$\bar{}$ = dimensional quantities
$\hat{}$ = estimated quantity

Introduction

INTERFACE conductance to heat flow between periodically contacting solids is of interest in many engineering applications including, among others, heat transfer between a valve and its seat in internal combustion engines, and cyclic manufacturing processes like plastic injection molding and die casting. Comprehensive reviews of the subject of interface conductance may be found in Refs. 1–3.

Experimental studies on the determination of contact conductance between periodically contacting solids under quasi-steady-state conditions can be found in Refs. 4–7. In these experiments, linear[4] and quadratic[5–7] spatial extrapolation of the measured temperatures were used to estimate the interface temperatures and heat flux, in order to estimate the interface conductance.

More recent theoretical work by Flach and Özişik[8] and by Beck[9] on the estimation of the contact conductance between periodically contacting solids as a problem of inverse analysis showed the inaccuracy of the extrapolation techniques. Flach and Özişik[8] used the integral transform technique together with a periodic B-Spline basis to represent the unknown interface temperatures, while Beck[9] used a combined parameter and function estimation.

The estimation problem becomes difficult when no prior information is available on the functional form of the interface conductance and when the period is very small, such as those situations encountered in internal combustion engines.

In the present work, the conjugate gradient method with adjoint equation is used to solve the inverse problem of estimating interface conductance between periodically contacting solids, with no prior information on the functional form of the unknown quantity. Alifanov et al.[10,11] were among the earlier users of this approach. More recently, the method has been used for solving the inverse problems of determining the wall heat flux in laminar flow through a parallel plate duct[12] and the interface conductance between mold and casting during solidification.[13] The method converges faster than the steepest descent method[14] and is capable of handling very small periods that cannot be handled accurately with other techniques.

Direct Problem

Figure 1 shows the geometry and the coordinates for the one-dimensional physical problem considered here. Two rods, referred to as regions 1 and 2, are contacting periodically with $\bar{h}(\bar{t})$ being the contact conductance at the interface. The noncontacting ends are kept at constant, but different temperatures $\bar{T}_{01}$ and $\bar{T}_{02}$. It is assumed that sufficient number of contacts has been made so that the quasi-steady-state condition is established for the temperature distribution in the solids, that is, the temperature distribution in the regions during one period is identical to that in the following period.

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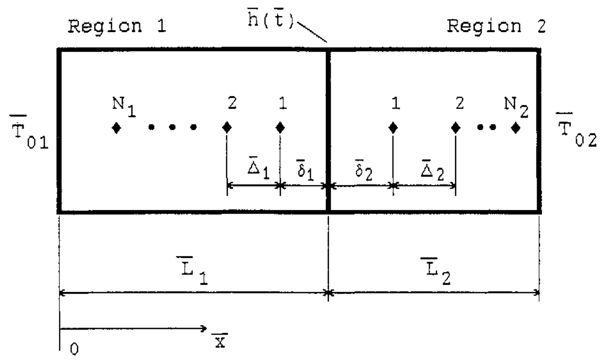

Fig. 1 Physical problem.

The mathematical formulation of this heat conduction problem is given in dimensionless form as

Region 1 ($0 < x < 1$)

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_1}{\partial t} \quad \text{in} \quad 0 < x < 1, t > 0 \tag{1a}$$

$$T_1 = 0 \quad \text{at} \quad x = 0, t > 0 \tag{1b}$$

$$-\frac{\partial T_1}{\partial x} = h(t)[T_1 - T_2] \quad \text{at} \quad x = 1, t > 0 \tag{1c}$$

$$T_1(x, 0) = T_1(x, \tau) \tag{1d}$$

Region 2 ($1 < x < 1 + L$)

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T_2}{\partial t} \quad \text{in} \quad 1 < x < 1 + L, t > 0 \tag{2a}$$

$$-K\frac{\partial T_2}{\partial x} = h(t)[T_1 - T_2] \quad \text{at} \quad x = 1, t > 0 \tag{2b}$$

$$T_2 = 1 \quad \text{at} \quad x = 1 + L, t > 0 \tag{2c}$$

$$T_2(x, 0) = T_2(x, \tau) \tag{2d}$$

where the following dimensionless quantities were defined:

$$x = (\bar{x}/\bar{L}_1) \tag{3a}$$

$$t = (\bar{\alpha}_1\bar{t}/\bar{L}_1^2) \tag{3b}$$

$$T = (\bar{T} - \bar{T}_{01}/\bar{T}_{02} - \bar{T}_{01}) \tag{3c}$$

$$h = (\bar{h}\bar{L}_1/\bar{K}_1) \tag{3d}$$

$$\alpha = (\bar{\alpha}_2/\bar{\alpha}_1) \tag{3e}$$

$$K = (\bar{K}_2/\bar{K}_1) \tag{3f}$$

$$L = (\bar{L}_2/\bar{L}_1) \tag{3g}$$

The direct problem considered here is concerned with the determination of the medium temperature, while the thermophysical properties, interface conductance $h(t)$, and the boundary conditions at the outer ends of the regions, are known.

Inverse Problem

For the inverse problem, the interface conductance $h(t)$ is regarded as being unknown, but everything else in the system of Eqs. (1–3) is known. In addition, temperature readings taken at some appropriate locations within the medium are considered available.

Referring to the nomenclature shown in Fig. 1, we assume that N_1 sensors are located in region 1 and N_2 sensors are located in region 2. The first sensors are located at distances δ_1 and δ_2 from the interface, while the remaining sensors are located with equal spacing of Δ_1 in region 1 and Δ_2 in region 2.

Let the temperature readings taken with these sensors over the period τ be denoted by

$$Y_{1i}(t) \equiv Y_{1i}, \qquad i = 1, 2, \ldots, N_1 \text{ in region 1}$$

$$Y_{2j}(t) \equiv Y_{2j}, \qquad j = 1, 2, \ldots, N_2 \text{ in region 2}$$

We note that the measured temperatures Y_{1i} and Y_{2j} contain measurement errors.

Then, the inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data Y_{1i} ($i = 1, 2, \ldots, N_1$) and Y_{2j} ($j = 1, 2, \ldots, N_2$), estimate the unknown $h(t)$ over τ.

It is assumed that no prior information is available on the functional form of $h(t)$, except that τ is known. We are after the function $h(t)$ over the whole time domain $[0, \tau]$, only with the assumption that $h(t)$ belongs to the space of the square integrable functions in this domain, i.e.

$$\int_0^\tau [h(t)]^2 \, dt < \infty$$

Since all the measured temperatures are used to compute the entire unknown function for one period of variation, the method used here may be classified as whole-domain.[15]

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[h(t)] = \int_{t=0}^{\tau}\left[\sum_{i=1}^{N_1}(T_{1i} - Y_{1i})^2\right] dt + \int_{t=0}^{\tau}\left[\sum_{j=1}^{N_2}(T_{2j} - Y_{2j})^2\right] dt \tag{4}$$

Here, T_{1i} and T_{2j} are the estimated temperatures in the regions 1 and 2, respectively, at the measurement locations. These quantities are determined from the solution of the direct problem given previously, by using an estimate $\hat{h}^k(t)$ for the exact $h(t)$.

Conjugate Gradient Method for Minimization

The following iterative process based on the conjugate gradient method[10–14,16] is now used for the estimation of $h(t)$ by minimizing the above functional $J[h(t)]$

$$\hat{h}^{k+1}(t) = \hat{h}^k(t) - \beta_k p^k(t) \quad \text{for} \quad k = 0, 1, 2, \ldots \tag{5a}$$

where β_k is the "search step size" in going from iteration k to iteration $k + 1$; and $p^k(t)$ is the "direction of descent" (i.e., search direction) given by

$$p^k(t) = J'^k(t) + \gamma_k p^{k-1}(t) \tag{5b}$$

which is a conjugation of the gradient direction $J'^k(t)$ at iteration k and the direction of descent $p^{k-1}(t)$ at iteration $k - 1$. The "conjugation coefficient" γ_k is determined from

$$\gamma_k = \frac{\int_{t=0}^{\tau}[J'^k(t)]^2 \, dt}{\int_{t=0}^{\tau}[J'^{k-1}(t)]^2 \, dt} \quad \text{for} \quad k = 1, 2, \ldots$$

with

$$\gamma_k = 0 \quad \text{for} \quad k = 0 \tag{5c}$$

We note that when $\gamma_k = 0$ for any k, in Eq. (5b) the direction of descent $p^k(t)$ becomes the gradient direction, i.e., the "steepest descent" method is obtained.

The convergence of the above iterative procedure to a local minimum of the functional given by Eq. (4) is guaranteed.[14]

To perform the iterations according to Eqs. (5), we need to compute the step size β_k and the gradient of the functional $J'^k(t)$. In order to develop expressions for the determination of these two quantities, a "sensitivity problem" and an "adjoint problem" are constructed as described below.

Sensitivity Problem and Search Step Size

The sensitivity problem is obtained from the original direct problem defined by Eqs. (1) and (2) in the following manner. It is assumed that when $h(t)$ undergoes a variation $\Delta h(t)$, T_1 is perturbed by ΔT_1 and T_2 is perturbed by ΔT_2. Then, replacing in the direct problem $h(t)$ by $h(t) + \Delta h(t)$, T_1 by $T_1 + \Delta T_1$ and T_2 by $T_2 + \Delta T_2$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problems for the sensitivity functions ΔT_1 and ΔT_2 are obtained:

Region 1 ($0 < x < 1$)

$$\frac{\partial^2 \Delta T_1}{\partial x^2} = \frac{\partial \Delta T_1}{\partial t} \quad \text{in} \quad 0 < x < 1, t > 0 \tag{6a}$$

$$\Delta T_1 = 0 \quad \text{at} \quad x = 0, t > 0 \tag{6b}$$

$$-\frac{\partial \Delta T_1}{\partial x} = \Delta h(t)(T_1 - T_2) + h(t)(\Delta T_1 - \Delta T_2)$$

$$\text{at} \quad x = 1, t > 0 \tag{6c}$$

$$\Delta T_1(x, 0) = \Delta T_1(x, \tau) \tag{6d}$$

Region 2 ($1 < x < 1 + L$)

$$\frac{\partial^2 \Delta T_2}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \Delta T_2}{\partial t} \quad \text{in} \quad 1 < x < 1 + L, t > 0 \tag{7a}$$

$$-K\frac{\partial \Delta T_2}{\partial x} = \Delta h(t)[T_1 - T_2] + h(t)[\Delta T_1 - \Delta T_2]$$

$$\text{at} \quad x = 1, t > 0 \tag{7b}$$

$$\Delta T_2 = 0 \quad \text{at} \quad x = 1 + L, t > 0 \tag{7c}$$

$$\Delta T_2(x, 0) = \Delta T_2(x, \tau) \tag{7d}$$

The functional $J(\hat{h}^{k+1})$ for iteration $k + 1$ is obtained by rewriting Eq. (4) as

$$J(\hat{h}^{k+1}) = \int_{t=0}^{\tau} \sum_{i=1}^{N_1} [T_{1i}(\hat{h}^k - \beta_k p^k) - Y_{1i}]^2 \, dt + \int_{t=0}^{\tau} \sum_{j=1}^{N_2} [T_{2j}(\hat{h}^k - \beta_k p^k) - Y_{2j}]^2 \, dt \tag{8a}$$

where we replaced $\hat{h}^{k+1}$ by the expression given by Eq. (5a). If temperatures $T_{1i}(\hat{h}^k - \beta_k p^k)$ and $T_{2j}(\hat{h}^k - \beta_k p^k)$ are linearized by a Taylor series expansion, Eq. (8a) takes the form

$$J(\hat{h}^{k+1}) = \int_{t=0}^{\tau} \sum_{i=1}^{N_1} [T_{1i}(\hat{h}^k) - \beta_k \Delta T_{1i}(p^k) - Y_{1i}]^2 \, dt + \int_{t=0}^{\tau} \sum_{j=1}^{N_2} [T_{2j}(\hat{h}^k) - \beta_k \Delta T_{2j}(p^k) - Y_{2j}]^2 \, dt \tag{8b}$$

where $T_{1i}(\hat{h}^k)$ and $T_{2j}(\hat{h}^k)$ are the solutions of the direct problems [Eqs. (1) and (2)], by using the estimate $\hat{h}^k(t)$ for $h(t)$. The sensitivity functions $\Delta T_{1i}(p^k)$ and $\Delta T_{2j}(p^k)$ are taken as the solutions of problems [Eqs. (6) and (7)], by setting $\Delta h(t) = p^k(t)$.

The search step size β_k is determined by minimizing the functional given by Eq. (8b) with respect to β_k. The following expression results:

$$\beta_k = \frac{\int_{t=0}^{\tau} \left[\sum_{i=1}^{N_1} (T_{1i} - Y_{1i})\Delta T_{1i} + \sum_{j=1}^{N_2} (T_{2j} - Y_{2j})\Delta T_{2j} \right] dt}{\int_{t=0}^{\tau} \left[\sum_{i=1}^{N_1} (\Delta T_{1i})^2 + \sum_{j=1}^{N_2} (\Delta T_{2j})^2 \right] dt} \tag{9}$$

where $T_{1i} \equiv T_{1i}(\hat{h}^k)$, $T_{2j} \equiv T_{2j}(\hat{h}^k)$, $\Delta T_{1i} \equiv \Delta T_{1i}(p^k)$, and $\Delta T_{2j} \equiv \Delta T_{2j}(p^k)$.

Adjoint Problem and the Gradient Equation

To obtain the adjoint problem, Eq. (1a) is multiplied by the Lagrange multiplier $\lambda_1(x, t)$, Eq. (2a) is multiplied by the Lagrange multiplier $\lambda_2(x, t)$ and the resulting expressions are integrated over the time and correspondent space domains. Then the results are added to the right side of Eq. (4) to yield the following expression for the functional $J[h(t)]$:

$$J[h(t)] = \int_{t=0}^{\tau} \left[\sum_{i=1}^{N_1} (T_{1i} - Y_{1i})^2 \right] dt + \int_{t=0}^{\tau} \left[\sum_{j=1}^{N_2} (T_{2j} - Y_{2j})^2 \right] dt + \int_{t=0}^{\tau} \int_{x=0}^{1} \lambda_1(x, t) \left(\frac{\partial^2 T_1}{\partial x^2} - \frac{\partial T_1}{\partial t} \right) dx \, dt + \int_{t=0}^{\tau} \int_{x=1}^{1+L} \lambda_2(x, t) \left(\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T_2}{\partial t} \right) dx \, dt \tag{10}$$

The variation ΔJ is obtained by perturbing T_1 by ΔT_1, T_2 by ΔT_2 in Eq. (10), subtracting from the resulting expression the original equation, Eq. (10) and neglecting the second-order terms. We thus find

$$\Delta J = \int_{t=0}^{\tau} \sum_{i=1}^{N_1} 2\Delta T_{1i}(T_{1i} - Y_{1i}) \, dt + \int_{t=0}^{\tau} \sum_{j=1}^{N_2} 2\Delta T_{2j}(T_{2j} - Y_{2j}) \, dt + \int_{t=0}^{\tau} \int_{x=0}^{1} \lambda_1(x, t) \left(\frac{\partial^2 \Delta T_1}{\partial x^2} - \frac{\partial \Delta T_1}{\partial t} \right) dx \, dt + \int_{t=0}^{\tau} \int_{x=1}^{1+L} \lambda_2(x, t) \left(\frac{\partial^2 \Delta T_2}{\partial x^2} - \frac{1}{\alpha}\frac{\partial \Delta T_2}{\partial t} \right) dx \, dt \tag{11}$$

In Eq. (11), the last two integral terms are integrated by parts; the initial and boundary conditions of the sensitivity problem given by Eqs. (6b–d) and (7b–d) are utilized and then ΔJ is allowed to go to zero. The vanishing of the integrands containing ΔT_1 and ΔT_2 leads to the following adjoint problem for the determination of $\lambda_1(x, t)$ and $\lambda_2(x, t)$:

Region 1 ($0 < x < 1$)

$$\frac{\partial^2 \lambda_1}{\partial x^2} + \frac{\partial \lambda_1}{\partial t} + \sum_{i=1}^{N_1} 2(T_{1i} - Y_{1i})\delta(x - x_i) = 0$$

$$\text{in} \quad 0 < x < 1, t > 0 \tag{12a}$$

$$\lambda_1 = 0 \quad \text{at} \quad x = 0, t > 0 \tag{12b}$$

$$\frac{\partial \lambda_1}{\partial x} = h(t)\left(\frac{\lambda_2}{K} - \lambda_1 \right) \quad \text{at} \quad x = 1, t > 0 \tag{12c}$$

$$\lambda_1(x, \tau) = \lambda_1(x, 0) \tag{12d}$$

Region 2 ($1 < x < 1 + L$)

$$\frac{\partial^2\lambda_2}{\partial x^2} + \frac{1}{\alpha}\frac{\partial\lambda_2}{\partial t} + \sum_{j=1}^{N_2} 2(T_{2j} - Y_{2j})\delta(x - x_j) = 0$$

$$\text{in} \quad 1 < x < 1 + L, t > 0 \tag{13a}$$

$$\frac{\partial\lambda_2}{\partial x} = h(t)\left[\frac{\lambda_2}{K} - \lambda_1\right] \quad \text{at} \quad x = 1, t > 0 \tag{13b}$$

$$\lambda_2 = 0 \quad \text{at} \quad x = 1 + L, t > 0 \tag{13c}$$

$$\lambda_2(x, \tau) = \lambda_2(x, 0) \tag{13d}$$

where $\delta(x - x_i)$ is the Dirac delta function. Here, x_i and x_j refer to the thermocouple locations in the regions 1 and 2, respectively.

Finally, the following integral term is left:

$$\Delta J = \int_{t=0}^{\tau}\left\{\left[\frac{\lambda_2(1, t)}{K} - \lambda_1(1, t)\right]\right.$$

$$\left.\cdot[T_1(1, t) - T_2(1, t)]\right\} \Delta h(t)\, \mathrm{d}t \to 0 \tag{14}$$

From the assumption that $h(t) \in L_2[0, \tau]$, ΔJ is related to the gradient $J'(t)$ by[10]

$$\Delta J = \int_{t=0}^{\tau} J'(t)\Delta h(t)\, \mathrm{d}t \tag{15}$$

A comparison of Eqs. (14) and (15) leads to the following expression for the gradient $J'(t)$ of the functional $J[h(t)]$:

$$J'(t) = \{[\lambda_2(1, t)/K] - \lambda_1(1, t)\}[T_1(1, t) - T_2(1, t)] \tag{16}$$

Stopping Criterion

If the problem contains no measurement errors, the traditional check condition specified as

$$J[h^{k+1}(t)] < \varepsilon \tag{17}$$

where ε is a small specified number, is used. However, the observed temperature data may contain measurement errors. Therefore, we do not expect the functional Eq. (4) to be equal to zero at the final iteration step. Following the experience of the authors,[10–13] we use the discrepancy principle as the stopping criterion, i.e., we assume that the temperature residuals may be approximated by

$$(T_{1i} - Y_{1i}) \approx (T_{2j} - Y_{2j}) \approx \sigma \tag{18}$$

where σ is the standard deviation of the measurements, which is assumed to be constant. The above assumption was also made by Tikhonov[17] in order to find the optimal regularization parameter. Substituting Eq. (18) into Eq. (4), the following expression is obtained for ε:

$$\varepsilon = (N_1 + N_2)\sigma^2\tau \tag{19}$$

Then, the stopping criterion is given by Eq. (17) with ε determined from Eq. (19).

Computational Procedure

The computational procedure for the solution of this inverse problem may be summarized as follows:

Suppose $\hat{h}^k(t)$ is available at iteration k.

Step 1: Solve the direct problem given by Eqs. (1) and (2) for $T_1(x, t)$ and $T_2(x, t)$.

Step 2: Examine the stopping criterion given by Eq. (17) with ε given by Eq. (19). Continue if not satisfied.

Step 3: Solve the adjoint problem given by Eqs. (12) and (13) for $\lambda_1(x, t)$ and $\lambda_2(x, t)$.

Step 4: Compute the gradient of the functional $J'(t)$ from Eq. (16).

Step 5: Compute the conjugation coefficient γ_k from Eq. (5c).

Step 6: Compute the direction of descent p^k from Eq. (5b).

Step 7: Set $\Delta h(t) = p^k(t)$ and solve the sensitivity problem given by Eqs. (6) and (7) for $\Delta T_1(x, t)$ and $\Delta T_2(x, t)$.

Step 8: Compute the search step size β_k from Eq. (9).

Step 9: Compute the new estimate for $\hat{h}^{k+1}(t)$ from Eq. (5a) and return to step 1.

Results and Discussion

The problems of periodically contacting surfaces involving very small periods are the most difficult on which to perform an inverse analysis. Therefore, to illustrate the accuracy of the conjugate gradient method under very strict conditions, we examine problems with very small periods.

Consider two identical regions each of length $\bar{L}_1 = \bar{L}_2 =$ 0.1 m and made of brass ($\bar{K}_1 = \bar{K}_2 = 106.1$ W/mK; $\bar{\alpha}_1 = \bar{\alpha}_2 = 3.4 \times 10^{-5}$ m²/s) studied experimentally in Ref. 5 and theoretically by using the B-spline method in Ref. 8. Each region contains four sensors and 18 measurements are made per sensor per period. Figure 1 shows the notation for the geometry, while Table 1 lists the dimensional and dimensionless sensor locations, as well as the periods for variation of $h(t)$ considered here.

Let $h(t)$ vary in the form

$$h(t) = \begin{cases} 2 & \text{for the contact period} \\ 0 & \text{for the noncontact period} \end{cases} \tag{20}$$

This dimensionless value of $h(t) = 2$ corresponds to a dimensional contact conductance of 2122 W/m²K, which is encountered in the contact of metallic wavy surfaces.[3] Both exact and inexact temperature measurements are considered, but all the other quantities used in the inverse analysis are assumed to be errorless.

Due to the periodic characteristic of the problem, it has been shown[18] that under the quasi-steady-state condition, the temperature distributions in the regions vary only within a finite depth of δ_T below the surface. The temperature distributions in each region, at the end of the contact and noncontact periods, for $\tau = 10^{-1}$, are presented in Fig. 2. There-

Table 1 Periods and sensor locations considered

Variable	Dimensionless	Dimensional
Period	10^{-1}	29.41 s
	10^{-2}	2.94 s
	10^{-3}	0.29 s
Sensor locations	$\delta = \Delta = 0.005$	$\bar{\delta} = \bar{\Delta} = 0.5$ mm
	$\delta = \Delta = 0.01$	$\bar{\delta} = \bar{\Delta} = 1$ mm
	$\delta = 0.05$; $\Delta = 0.1$	$\bar{\delta} = 5$ mm; $\bar{\Delta} = 10$ mm

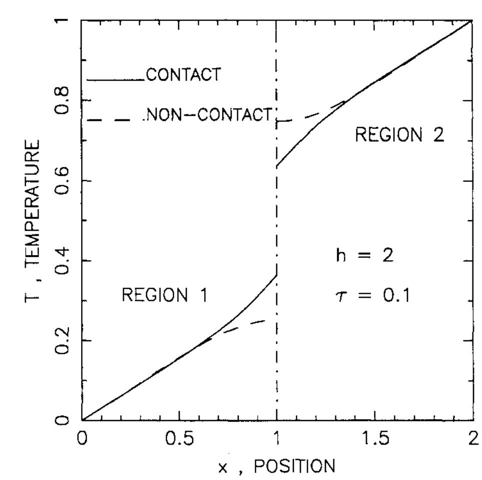

Fig. 2 Temperature distribution on the regions.

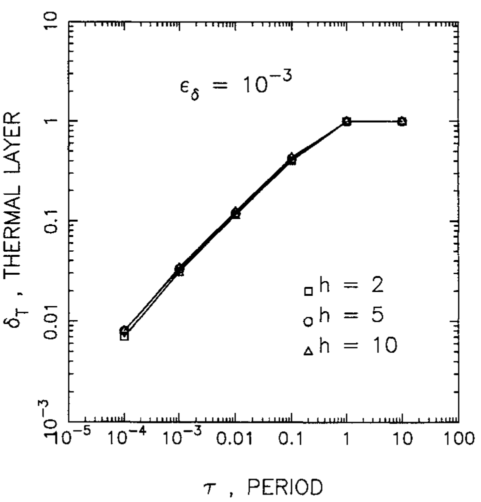

Fig. 3 Effects of τ and h on the thermal layer.

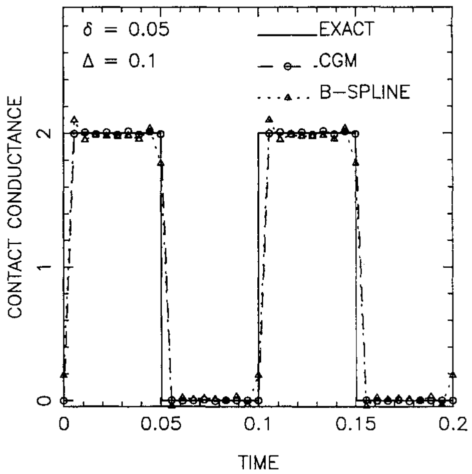

Fig. 4 Inverse solution for exact measurements and $\tau = 10^{-1}$.

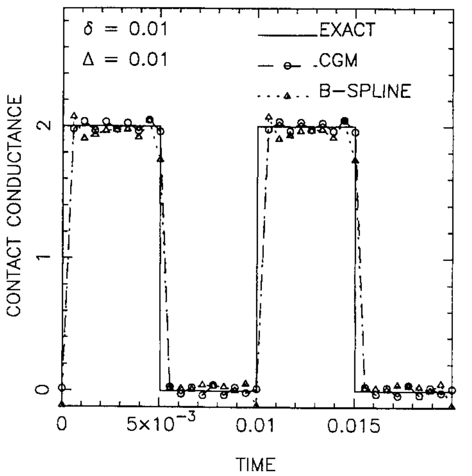

Fig. 5 Inverse solution for exact measurements and $\tau = 10^{-2}$.

fore, if the sensors are located outside this thermal layer δ_T, no difference can be detected between the temperature measurements for the contact and noncontact periods. Thus, to obtain meaningful results from the temperature measurements, the sensors must be located within the thermal layer δ_T. Here, we define δ_T as the depth below the surface such that

$$|T_{\tau/2} - T_\tau| < \varepsilon_\delta \tag{21}$$

where ε_δ is a fixed tolerance.

Figure 3 shows the effects of the period τ and the contact conductance h during the contact period on the dimensionless thickness δ_T of the thermal layer for a tolerance $\varepsilon_\delta = 10^{-3}$. We note that the value of the contact conductance has a negligible effect on δ_T; on the other hand, δ_T is strongly dependent on the period τ such that δ_T becomes very small for short periods. For example, for the physical case considered here, for $\tau = 10^{-4}$ (i.e., 0.029 s) the thermal layer is of the order of 10^{-3}, which corresponds to a thickness of tenths of a millimeter.

The results presented in Fig. 3 are obviously dependent on the tolerance ε_δ, which is directly related to the accuracy of the sensors used. Therefore, these results are just a qualitative indication of the behavior of the thermal layer, with respect to variations in h and τ.

In order to compare the results of the inverse analysis based on the conjugate gradient method (CGM) considered here and the B-spline method described in Ref. 8, we present in Figs. 4–9 a comparison of the estimated and exact values of the contact conductance obtained by both methods, for periods ranging from $\tau = 10^{-1}$ to 10^{-3}. The sensor locations were chosen so that all the sensors would be inside the thermal layer given in Fig. 3. It is to be noted that by locating the sensors very close to the surface we are also neglecting the effects of heat flux constrictions.[3]

In Figs. 4 to 6, the measurements were considered errorless, i.e., just the deterministic component of the error is involved on the solution. For such a case, the estimation by the conjugate gradient method of the functional form of the contact conductance is more accurate than that by the B-spline method. The B-spline solution exhibits oscillations near the discontinuities which increase with decreasing period. On the other

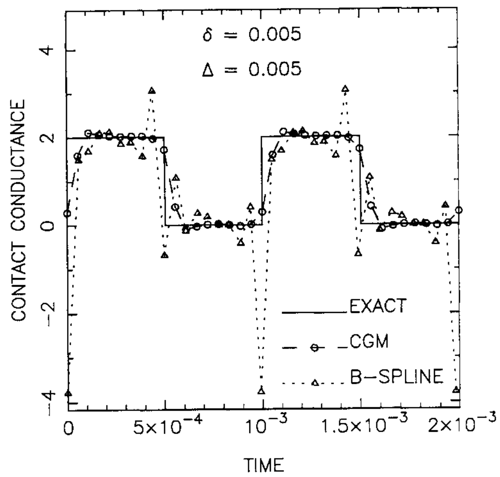

Fig. 6 Inverse solution for exact measurements and $\tau = 10^{-3}$.

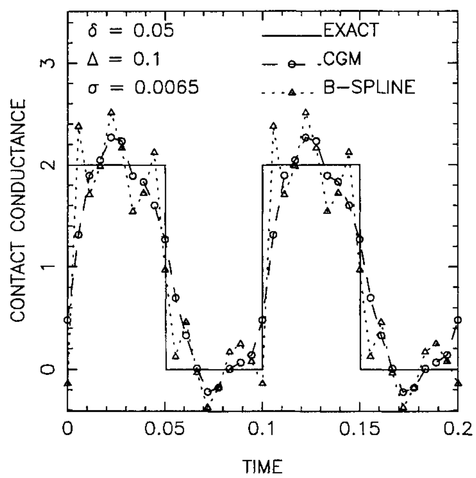

Fig. 7 Inverse solution for $\sigma = 0.0065$ and $\tau = 10^{-1}$.

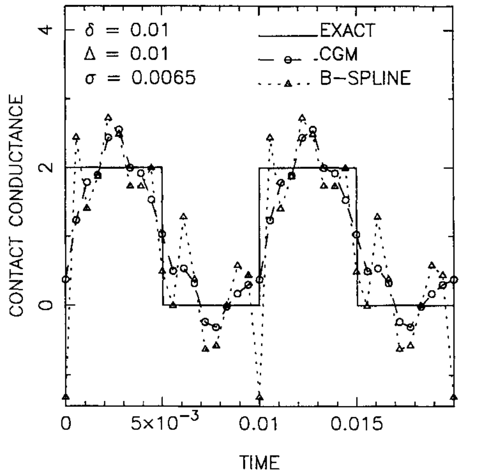

Fig. 8 Inverse solution for $\sigma = 0.0065$ and $\tau = 10^{-2}$.

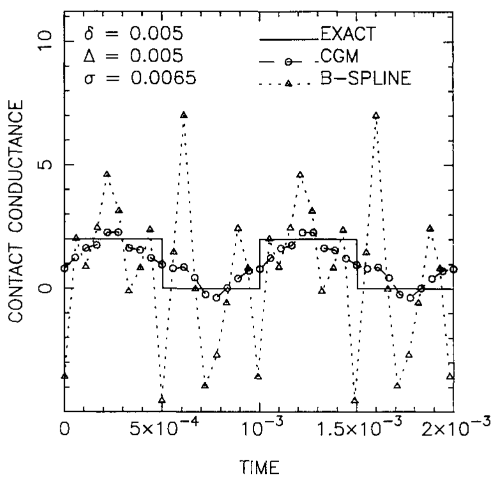

Fig. 9 Inverse solution for $\sigma = 0.0065$ and $\tau = 10^{-3}$.

hand, the solution with the conjugate gradient method is very stable and does not exhibit oscillations.

In order to compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors, with zero mean and constant standard deviation and simulate the inexact measurement data Y as

$$Y = Y_{exact} + \omega\sigma \tag{22}$$

where Y_{exact} is the solution of the direct problem with a known $h(t)$; σ is the standard deviation of the measurements; and ω is a random variable with normal distribution, zero mean and unitary standard deviation. The variable ω is generated by the subroutine DRNNOR of the IMSL.[19]

The results obtained for the same cases considered previously are presented in Figs. 7–9 for $\sigma = 0.0065$. For the 99% confidence level, this value of the standard deviation corresponds to errors up to 1.7% of the maximum temperature drop across the regions. The estimation of the interface conductance obtained with the conjugate gradient method and with the B-spline approach are of comparable accuracy for sufficiently large periods, as illustrated in these figures. However, the accuracy of both methods is reduced as the period decreases, but, for such cases, the conjugate gradient method is much more accurate than the B-spline approach.

The total root mean square (rms) error computed from

$$\text{rms } e_{total}(h) = \sqrt{\frac{1}{18}\sum_{i=1}^{18}[\hat{h}(t_i) - h(t_i)]^2} \tag{23}$$

Table 2 Total rms error and relative error

Period	rms e_{total}, h		e/T_{var}
	B-Spline	CGM	
10^{-1}	0.36	0.36	0.15
10^{-2}	0.60	0.43	0.42
10^{-3}	2.90	0.58	1.67

Table 3 Total rms error for $\tau = 10^{-1}$

Sensor location	rms e_{total}, h
$\delta = 0.05$; $\Delta = 0.1$	0.36
$\delta = \Delta = 0.01$	0.17
$\delta = \Delta = 0.005$	0.16

is presented in Table 2 for the cases considered in Figs. 7–9. Clearly, the rms error increases as the period decreases and, for $\tau = 10^{-3}$, the rms error for the B-spline method is much higher than that for the conjugate gradient method. The increase in the rms error with decreasing period is expected. This is due to the fact that for shorter periods, the measurement error increases relatively to the maximum temperature variation in the regions. This maximum variation occurs at the contacting interface (see Fig. 2) and gives the upper limit of the temperature variations the sensors will measure. The ratio between e at the 99% confidence level for $\sigma = 0.0065$ and T_{var} is presented in the last column of Table 2.

Table 3 shows the total rms error computed from Eq. (23) for $\tau = 10^{-1}$ and for three different sensor locations. We note that the rms error is reduced as the sensor is located near the contacting surface. This result is expected, because sensors located closer to the interface loose less information due to the diffusive character of the problem.

Conclusions

The conjugate gradient method with adjoint equation was successfully applied for the solution of the inverse problem to determine the interface conductance between periodically contacting surfaces, under quasi-steady-state conditions.

The comparison of the results obtained with the conjugate gradient method with those obtained with the B-spline approach, reveals that the conjugate gradient method is superior when short periods are considered. For large periods, the two methods appear to be of comparable accuracy. However, for extremely small periods, both methods tend to fail, because the temperature measurement errors become of the same order of magnitude as the maximum temperature variation inside the thermal layer.

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